Application of Options Theory to

Value Mining Flexibility

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ABSTRACT

Valuation techniques used in mining include technical analysis techniques and discounted cash flow (DCF) techniques. There is considerable disagreement as to the best method to handle risk and uncertainty, leading to an array of valuation methods. One of the major disadvantages of traditional valuation methods is the inability to value management flexibility in the event of improved knowledge resolving uncertainty. Real options analysis is one way of valuing flexibility.

Real options may be analysed using exact formulae, numerical approximation, or Monte Carlo simulation. Numerical approximation using a binomial analysis technique offers the flexibility to handle complex real options found in mining valuation.

Early options analysis was performed on the basis of an underlying commodity price exhibiting an exponential growth. Models are available, however, to base analyses on a price that reverts to some mean value over time.

An example of a complex compound option involving decisions on grade control, abandonment, mothballing and expansion was developed by Winsen, 1994. This model has been further enhanced and forms the basis for the analysis in this paper.

The Winsen model did not address valuation of a mine with limited mining reserves, or the consideration of the impact of the cut-off grade on mine value. Both of these cases are considered in this paper. The algorithm developed to value these cases incorporates path dependence, linear programming, and three-staged iterative dynamic programming. Further work is suggested to turn these concepts into a practical valuation tool.

INTRODUCTION

The uses for financial valuation of mines

A common aspect of mining management decision making is the need to place a value on mining related activities. These decisions may be categorised as (Park and Herath, 2000), (Lawrence, 2001):

- Equipment and process selection.
- Equipment replacement.
- New mine investment and production expansion.
- Cost reduction.
- Service improvement.
- In support of capital raising or in defence of mergers and takeovers as required under Australian corporations law as part of an independent expert report.
- Revaluation of existing assets for inclusion in current cost accounting balance sheets as determined by the International Accounting Standards Board using the new International Valuation Standards Guidance Note for the extractive industries formulated in 2003 (Heffernan,2004).

Traditional approaches to the valuation of mines

Two approaches are common: the technical analysis techniques; and discounted cash flow methods (Lawrence, 2001).Technical analysis techniques include:

- Yardstick methods where a rule of thumb such as price/earnings ratio or price/resource or production unit is used a basis for valuation.
- Joint venture (JV) terms method that takes into account existing JV agreements for the same or similar tenements.
- Comparability of sales method, otherwise known as real estate based valuation.
- Multiples of exploration expenditure where relevant and effective exploration expenditure is multiplied by a prospectivity factor of between 0 and 5.
- Geoscience rating method utilising a points based scoring system based on the prospectivity of the resource. Value is assigned based on the total point score.

Discounted cash flow (DCF) techniques require the estimation of the net present value (NPV) of the future stream of cash flows. These cash flows are discounted at an appropriate rate. There is considerable disagreement on the treatment of risk and the selection of the discount rate. Some alternative approaches include:

- Selection of a risk adjusted discount rate using the capital asset pricing model (CAPM) and the weighted average cost of capital (WACC) to estimate the impacts of gearing (Ballard, 1994). An aspect that seems to be ignored is the increased risk to equity when gearing is employed.
- Recognition of the shortfalls of the CAPM but use it in the absence of a better technology (O'Connor and McMahon, 1994). They prefer to give a range of values for different discount rates to show sensitivity to risk.
- Uncertainties in mineral valuation are too project specific to be assessed using the CAPM (Runge, 1994). Runge suggests varying the discount rate based on project specific criteria such as mine life, 'fit' with current operations, flexibility, and risk criteria.
- Presentation of discrete sensitivities is a better alternative to Monte-Carlo simulation due to the impracticality of this technique (Butler, 1994). In particular, Butler prefers the use of Bayesian statistical techniques to determine a downside value from a banker's perspective.
- NPV is only one input to value and strategic interests should not be ignored (O'Connor and McMahon, 1994). Adjustments should be made to the NPV for market premium, hedging, reserves, start-up capital, controlling interest, risk (geographic, political, currency), and synergy with other assets.
- Managerial flexibility should be incorporated into valuation analysis since management does not blindly follow a fixed plan irrespective of changes in commodity prices (Runge, 1994), (Lonergan, 1994) and (Winsen, 1994). Runge suggests adjusting the discount rate, while Lonergan and Winsen prefer the adoption of real options pricing techniques.
- Sorentino (2000) argues for the use of stochastic discounted cash flow analysis and Bayesian analysis to model the uncertainties inherent in valuation of early stage mining projects. He argues against the use of risk adjusted discount rates that take into account project uncertainties on the basis that these uncertainties are evaluated explicitly in the valuation technique.

Problems arising from traditional approaches – the value of flexibility

The primary shortfall of traditional NPV approaches is that they do not value management flexibility and organisational knowledge enhancement after the commitment of resources (Anderson, 2000). Risk is due to uncertainties and environmental changes and is fundamental for the development of new strategic options. Option pricing can be used to analyse these strategic decisions.

A number of techniques are available for the valuation of strategic projects (Luehrman, 1997). Different valuation methods are required for different problem types. These he splits into operations (assets in place), opportunities (real options) and equity claims. Valuation methodologies for operations include multiples of sales, book value, EBIT and cash flow as well as WACC-based DCF and Monte Carlo simulation. He recommends an adjusted present value (APV) technique where each cash flow is discounted at the correct rate rather than trying to calculate a WACC that applies to an average cash flow.

Valuation methodologies for opportunities include multiples for installed base and customer-subscriber, Bayesian decision trees, scenario analysis, Monte Carlo simulation, and option pricing. Luehrman (1997) recommends simple option pricing to gain an insight into the value of the opportunity. Park and Herath (2000) also argue that real options are more suited where the potential exists to delay investment since this technique emphasises the potential for creating value by resolving uncertainty.

Valuation methodologies for equity claims include net income multiples, P/E ratios, WACC-based DCF (minus debt), scenario analysis and Monte Carlo simulation. Luehrman (1997) recommends the use of equity cash flow (ECF) as a comparison technique. ECF evaluates the value of an equity claim on a bundle of assets and opportunities.

REAL OPTIONS ANALYSIS

Real options may be evaluated using:

- ❒ Exact valuation formulas based on solutions to partial differential equations (Hull, 1993, p207ff).
- ❒ Binomial analysis techniques based on discrete time approximations of the above equations.
- ❒ Bayesian decision trees (Herath and Park, 2001).
- ❒ Monte Carlo simulation (Hull, 1993, p329ff).
- ❒ Combinations of the above techniques.

Binomial analysis techniques

Park and Herath (2000) note the restrictive assumptions involving asset price dynamics inherent in the Black and Scholes model. They suggest that the use of the binomial model is more feasible because of the complexity of real life capital budgeting, the potential for multiple variables and the absence of a risk free hedge such that arbitrage pricing may not hold.

With the binomial analysis technique a binomial lattice of prices is first constructed (Hull, 1993, p335ff). At time zero the price S is known. For each successive period, Δt, the price may move up by a factor u or down by a factor d. At time $i\Delta t$ the price may be expressed as,

$$Su^j d^{i-j} \quad for\ j = 0\ to\ i$$

where,

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = 1/u$$

σ^2 = price variance

Δt = time increment

$i\Delta t$ = time period

Options are evaluated using dynamic programming, starting at the last time increment and working backwards through the tree to derive the option value. The terminal node values in the last time increment $i = N$ are given by,

$$f_{N,j} = \max(S_{N,j} - X, 0) \quad for\ j = 0\ to\ N$$

for a call option and,

$$f_{N,j} = \max(X - S_{N,j}, 0) \quad for\ j = 0\ to\ N$$

for a put option,

where,

X = the exercise price.

For European options where no early exercise is possible, the node values for earlier time periods are,

$$f_{i,j} = e^{-r\Delta t}\left[pf_{i+1,j+1} + (1-p)f_{i+1,j}\right] \quad for\ j = 0\ to\ i\ and\ i = 1\ to\ N-1$$

where,

$p = \dfrac{e^{r\Delta t} - d}{u - d}$ is the risk neutral probability of an upward price movement.

For an American option early exercise is possible and the nodal values are given by,

$$f_{i,j} = \max\left\{Su^j d^{i-j} - X, e^{-r\Delta t}\left[pf_{i+1,j+1} + (1-p)f_{i+1,j}\right]\right\} \quad for\ j = 0\ to\ i\ and\ i = 1\ to\ N-1$$

for a call option and,

$$f_{i,j} = \max\left\{X - Su^j d^{i-j}, e^{-r\Delta t}\left[pf_{i+1,j+1} + (1-p)f_{i+1,j}\right]\right\} \quad for\ j = 0\ to\ i\ and\ i = 1\ to\ N-1$$

for a put option.

Uses of real options analysis

Trigeorgis (1993) describes the common types of real options as:

- ❒ Option to defer an investment, waiting to see if prices justify development. Valued as a call option.

- Staged development that allows abandonment mid-stream if new information is unfavourable. Valued as a compound option.
- Option to alter output by expanding, contracting or temporary shutdown.
- Option to abandon permanently and realise the resale value of capital assets.
- Option to switch inputs, processes or outputs as prices and costs vary.
- Growth options where an early investment is a prerequisite in a chain of interrelated projects, opening up future growth opportunities.
- Multiple interacting options are typical of real life situations. The combined option value differs from the sum of the individual options. Real options also interact with financial options.

Complex, interacting real-life options require numerical techniques such as Monte Carlo simulation, Bayesian decision trees or binomial lattices.

PRICING ASSUMPTIONS IN REAL OPTIONS ANALYSIS

The basis for the early work on real options is geometric Brownian motion. The expected value for price at time "t" can be shown to be (Robel, 2001),

$$E[S_t] = S_0 e^{\mu t}$$

and the variance as,

$$Var[S_t] = S_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right).$$

This price model therefore describes exponential growth under uncertainty.

Where a real option is dependant on a commodity price Robel (2001) and Laughton and Jacoby (1993) argue that the price may be subject to random shocks, but that in the long run, competitive pressures will ensure that it reverts to a long term sustainable level. This situation describes most mining ventures where the investing company is a price-taker in the commodity market.

The impact of price reversion is to reduce the uncertainty in long-term revenues. Laughton and Jacoby (1993) found that use of a single discounted rate to value projects with unequal lives and price reversion introduces a bias against long-term investments.

Mean reversion of prices

Calistrate (cit. Robel, 2001) suggested the following binomial approximation to the risk-neutralised mean-reverting process. The price tree is calculated as described previously. The probability of an up step at time *i*, after *j* up steps is calculated from,

$$P_{i,j} = P\left(\frac{1}{2}\left[1 + \frac{\sqrt{\Delta t}}{\sigma}\mu_{i,j}^{*}\right]\right)$$

where,

$$\mu^*_{i,j} = \frac{\lambda\left[\bar{S} - S_{i,j}\right]}{S_{i,j}} - \rho\sigma\phi - \frac{\sigma^2}{2}$$

and where,

$$P(x) = \begin{cases} 0, & if\ x > 0; \\ x, & if\ 0 \le x \le 1; \\ 1, & if\ x > 1. \end{cases}$$

$\rho =$ correlation coefficient between the market and commodity price Wiener processes.

$\phi = \frac{\mu_m - r}{\sigma_m}$ is the market cost of risk.

A NEW VALUATION ALGORITHM FOR CASES WITH LIMITED RESOURCES

The gold mining example provided in the paper to the Valmin conference in 1994 by Winsen has been re-worked to provide a valuation algorithm for two common mining problems. The original example valued a gold mine that could mine from one of three areas in any year. Sufficient reserves existed in each area to ensure that a lack of reserves would not occur within the proposed mine life for any mining area. I have extended this model to cover two alternate mining scenarios:

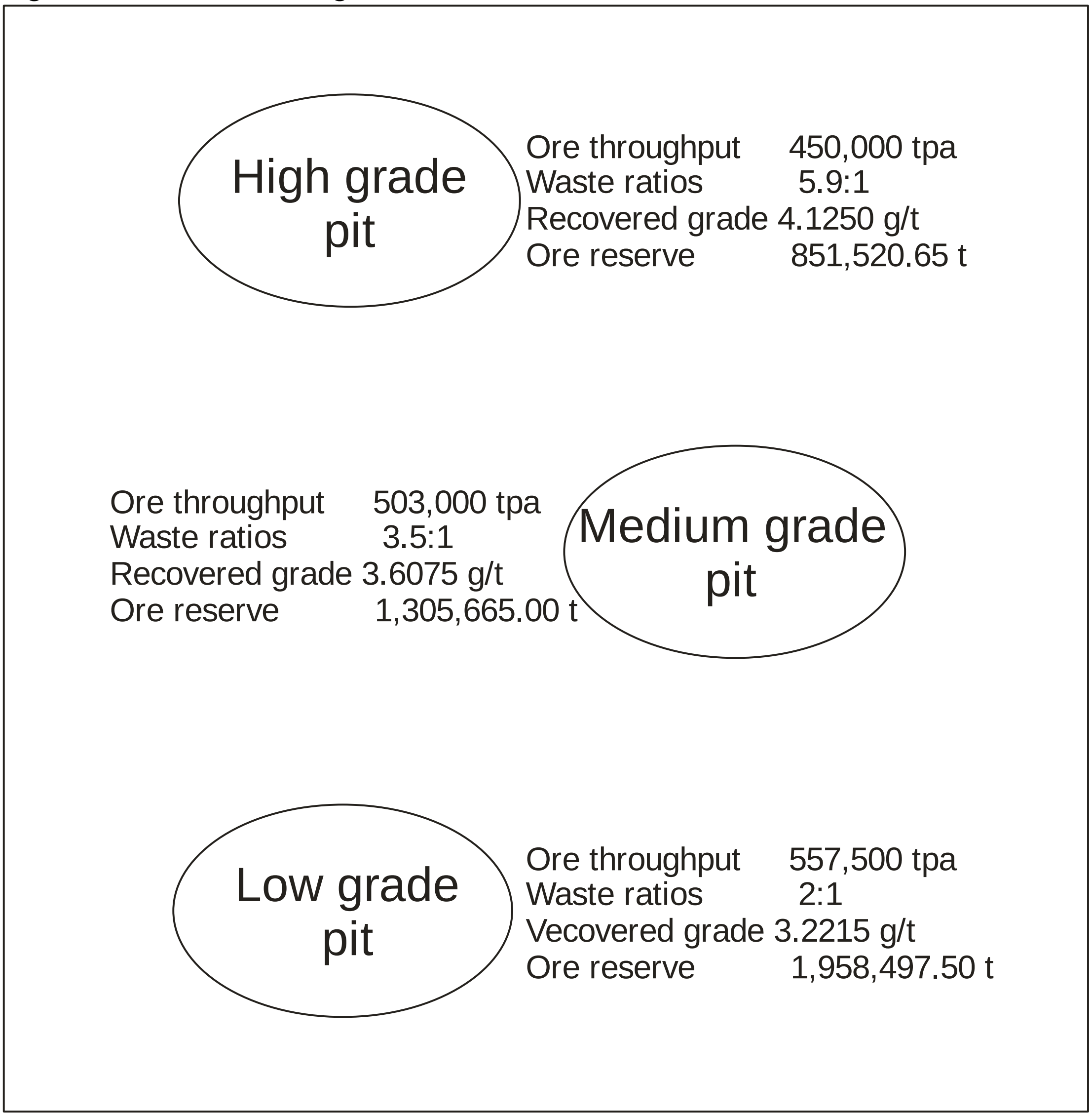
1. **Limited Mining Reserves.** In this case the mine life is extended from five years to eight years. This ensures that the three mining areas will run out of reserves within the mine life. Management options are to select the pits to be mined, whether to close the mine or to mothball for each time period.
2. **Cut-off Grade Problem.** In this case the three mining reserve quantities are assumed to represent the total reserves to three cut-off grades. That is, management may choose between processing high, medium or low head grades. Processing of a higher head grade results in some potential resource being sent to waste. Management also has the choice of mothballing or abandonment in each time period.

The original problem was solved by using a binomial technique for options valuation. This used dynamic programming to determine the maximum value for the mine.

Limited mining reserves

With limited mining reserves, the reserve available for mining is dependant on the amount mined in previous years. The mine to be valued is shown diagrammatically in ***Figure 1 – Limited Mining Reserves Problem.*** Three mining areas, with different quantities of ore and waste, can be mined in any order. Insufficient reserves exist to mine from the one area for the full eight years. Total reserves, however, are sufficient to last the full eight years.

Figure 1 – Limited Mining Reserves Problem.



The algorithm used to solve the unlimited reserves case cannot be used to value the limited mining reserves case because of the following limitations:

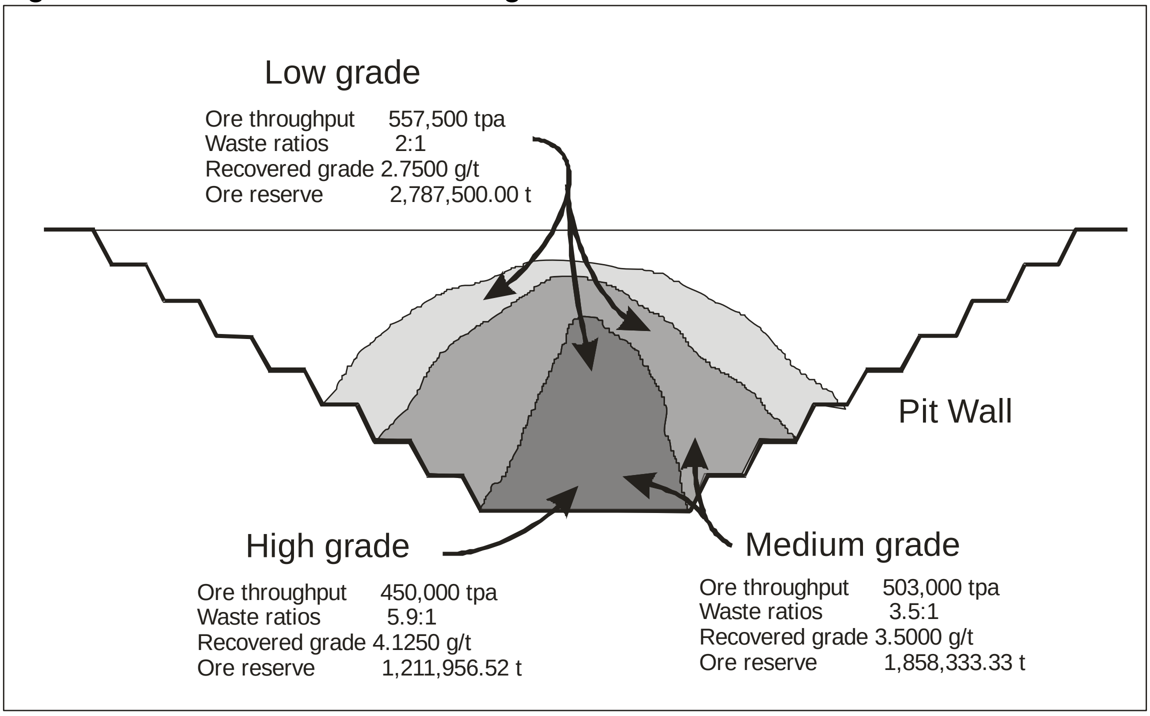
- Mining of differing reserve areas results in differing reserve quantities being available for mining in later years. This path dependence results in the binomial tree becoming a binary tree for a generalised algorithm.
- The selection of the optimum mix of mining from the high, medium and low grade pits at each node in the binomial (binary) tree is actually a linear programming problem. This becomes important in conditions where available reserves become a limitation on mining.
- The current dynamic programming formulation cannot be applied to the limited mining reserves case since the state of each stage in the current formulation does not optimise the decisions at the current stage without checking the feasibility of previous stages. This is essential for a dynamic programming algorithm (Taha, 1976, p208ff).

This requires that a new algorithm be developed to determine the maximum value for this compound option. A discussion of this algorithm and the shortfalls evident in the original algorithm for solving this new problem is given later in this paper.

Cut-off grade problem

Many practical mining problems relate to selecting the optimum grade for mining and processing. All ore lower than the cut-off is removed as waste and not processed for contained mineral. Any low grade material sent to waste is not available for processing at a later stage, due to its low incremental grade and due to burial in a waste dump. The mine to be valued is shown diagrammatically in ***Figure 2 – Pit Cross Section Showing Cut off Grade Problem*** Three mining cut-off grades are available. The grades and reserve tonnages indicated are an average for all ore at the respective cut-off grade (high, medium, or low). Reserves will be exhausted within the five year lease period. The mine life may last from 2.7 years for high grade mining, to 5 years for low grade. The cut-off grade may be varied at any time during the mine life.

Figure 2 – Pit Cross Section Showing Cut off Grade Problem



The solution to this problem is similar to the limited reserves problem and uses the same algorithm.

Calculation algorithm

All calculations are described in detail in the Appendix. As noted earlier, the base case algorithm has three major shortfalls when being applied to the limited reserves and cut-off grade problems.

Path dependence

Selection of the optimal reserve for mining at any decision node in the binomial tree results in reduced reserve quantities being available for mining in subsequent years. In the limited reserves case, only the reserve from the mined pit is reduced, but in the cut-off grade case the total reserve is reduced for all mining grades. The amount by which the total reserve is reduced is dependant on the grade selected for mining at the decision node.

Since earlier mining grade decisions will influence the future reserves available for mining and therefore future mining decisions, this introduces path dependence into the algorithm. This path dependence results in the binomial tree becoming a binary tree for a generalised algorithm.

Linear programming

The second limitation of the original algorithm relates to the method of determining the optimum grade to be mined at each decision node. The original algorithm assumes that the best solution is to mine only one ore type. This will be correct with unlimited reserves. As reserves become a constraint on production, the best solution may be to mine from two or more grades. Thus the decision at any node may be expressed as a linear optimisation problem. For the limited reserves case the linear optimisation problem may be expressed as,

$$\text{maximise,} \quad \mathrm{V'}_{a,b} = \sum_{j=1}^{3} Q_j \left(\frac{S_{a,b} x_j}{1+y} - (\lambda_j v_1 + v_2) \frac{(1+i)^a}{1+r} \right) c_j ,$$

$$\text{subject to,} \quad \sum_{j=1}^{3} c_j \leq 1,$$

$$c_j Q_j \leq R_{j,a,b}, \quad for \quad j = 1 \; to \; 3$$

$R_{j,a,b}$ is the remaining reserve of grade *j* in period *a* and for decision node *b*. In this formulation the proportion of grade *j* mined in a period is given by c_j.

For the cut-off ratio case mining to any cut-off ratio reduces the total remaining reserve. This will change the linear optimisation problem to,

$$\text{maximise,}\quad V'_{a,b} = \sum_{j=1}^{3} Q_j \left(\frac{S_{a,b} x_j}{1+y} - \left(\lambda_j v_1 + v_2\right) \frac{\left(1+i\right)^a}{1+r} \right) c_j ,$$

$$\text{subject to,}\quad \sum_{j=1}^{3} c_j \le 1,$$

$$\sum_{k=1}^{3} \begin{cases} 0 & , R_{k,a,b} = 0 \\ \dfrac{c_k Q_k R_{j,a,b}}{R_{k,a,b}} & , otherwise \end{cases} \le R_{j,a,b}, \quad for \quad j = 1\, to\, 3$$

The linear optimisation formulation above only optimises the variable costs and revenues for the various mining grade options. The recursive dynamic programming formulation still needs to take into account fixed costs, future values and the options of mothballing or abandoning the mine. The formula is,

$$V_{a,b} = MAX\left[\frac{pV_{a+1,b} + (1-p)V_{a+1,b+1}}{1+r} + MAX\left\{ \left(V'_{a,b} - F_1 (1+i)^{a-1}\right)(1-t), -F_2 \right\}, A \right]$$

The mine valuation calculation contains a linear programming optimisation function (*LINPROG(target,constraint,maximin)*) to allow solution of the linear optimisation problems formulated above. The function is written in VBA and has been adapted from a basic program developed by Stanford GSB (1974). The program uses a modified simplex method as described by Taha (1976, pp 47-72). Code for this function is shown in the Appendix.

Dynamic programming

Dynamic programming is described by Wagner (1969, p343) as an approach to optimisation consisting of the following structure:

- Decision variables and constraints are grouped according to stages which are considered sequentially.
- A state variable, the value being optimised, is carried forward from previous stages.
- The current decision influences the outcome of the next stage.
- Optimality of the current decision is based on its economic impact on the current and future stages.

The calculations are therefore recursive. In the base run algorithm the recursive calculations start at the final period and progress to the first period. With the limited reserves and cut-off ratio cases the recalculation of reserves based on mining decisions provides a feedback loop. The presence of a feedback loop contravenes the last condition of optimality. Calculation of the optimum value for a stage is not final since the feedback loop can change the reserves available for mining in earlier stages, thereby allowing a change in the value from the previous “optimum”.

The revised calculation algorithm

A new dynamic programming algorithm has been developed to calculate the maximum value for the new cases being studied. This algorithm is shown graphically in ***Figure 3 – Revised Calculation Algorithm***. It consists of nested optimisation algorithms. Linear programming is used at each decision node as described previously. A backwards calculation is used to calculate value and a forwards calculation is used to give reserves available for mining. This requires an iterative procedure that converges to a solution. These three calculation processes are allowed to iterate until a stable solution is attained. A final forwards calculation is used to test all possible solutions for the initial node and select the optimal solution for this node. The whole procedure is applied recursively for each period in turn until an optimal solution is reached.

The algorithm was programmed using VBA for use with an Excel spreadsheet. A description of the spreadsheet is attached in the appendix.

Figure 3 – Revised Calculation Algorithm

The calculation algorithm consists of three types of decision node connected in a binary tree. The decision nodes are:

- An initial node for which mineable reserves are defined and value is dependant on successive nodes;
- A calculation node which depends on preceding nodes for available reserves and succeeding nodes for determining value; and,
- A terminal node which depends on preceding nodes for available reserves, but for which an optimum value does not depend on any other nodes.

The reserves at each node are initially set equal to $R_{1,1}$. Linear programming is used at terminal nodes and an assumed solution at other nodes to select the mined quantities. A backwards recursive calculation is used to calculate the initial value $V_{1,1}$.(**Step 1**). Since the remaining reserves are impacted by the mining carried out at each node, the remaining reserves for mining are calculated using a recursive algorithm (**Step 2**). **Step 1** and **Step 2** are iterated until a stable solution is found.

The selected mining reserve at the initial node is then made sub-optimal, thus forcing the selection of an alternate reserve. The calculation again iterates between **Step 1** and **Step 2** until a stable $V_{1,1}$ is reached. This process is repeated until all alternative period 1 mining reserves have been evaluated. The reserve that gave the highest $V_{1,1}$ is then selected as optimal for period 1.

The process is then repeated with the node at period 1 removed from the optimisation calculation. The two period 2 nodes then become the initial nodes. **Step 3** shows the value calculation and **Step 4** shows the reserve calculation for the optimisation of period 2 nodes.

The procedure is repeated for each period until the nodes for the second last period are optimised. This is shown in **Step 5** and **Step 6**. A recursive calculation of value based on the optimal mining reserves for each period will then result in the maximum value for the mine, $V_{1,1}$.

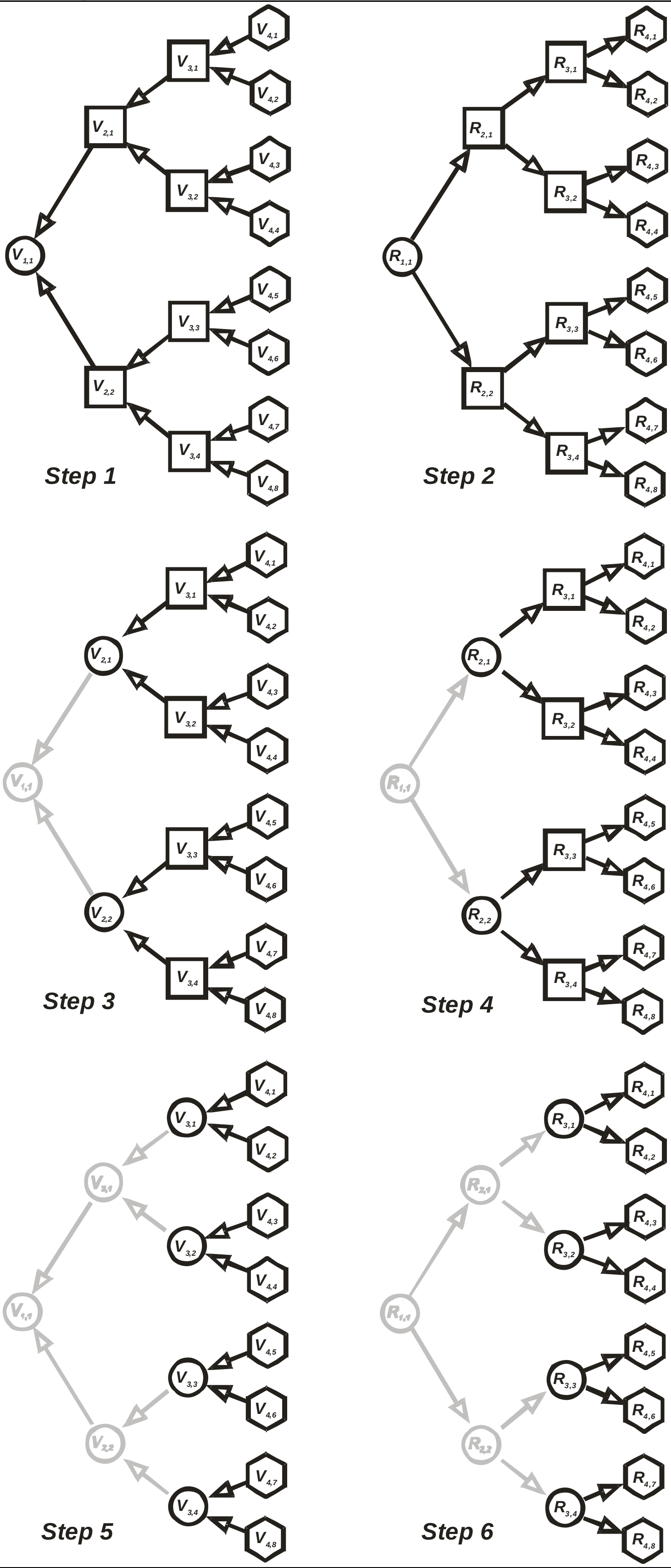


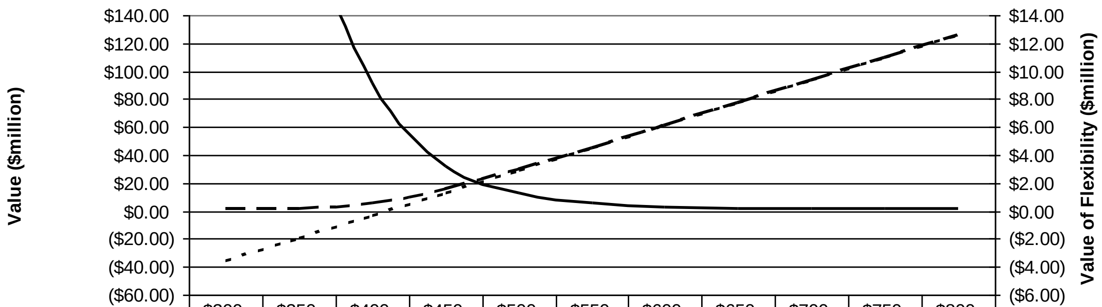
Figure 3 - Revised Calculation Algorithm

THE VALUE OF MANAGEMENT FLEXIBILITY

The real option value was determined for the limited mining reserves and cut-off grade problems outlined above. The real options value for the limited reserves case is shown in ***Figure 4 – Real Option Value – Limited Mining Reserves Problem*** and the value for the cut-off grade case in ***Figure 5 – Real Option Value – Cut-off Grade Problem***. A range of gold prices was used to show the variation of management flexibility with commodity price.

Figure 4 – Real Option Value – Limited Mining Reserves Problem

Figure 4 - Real Option Value - Limited Mining Reserves Problem



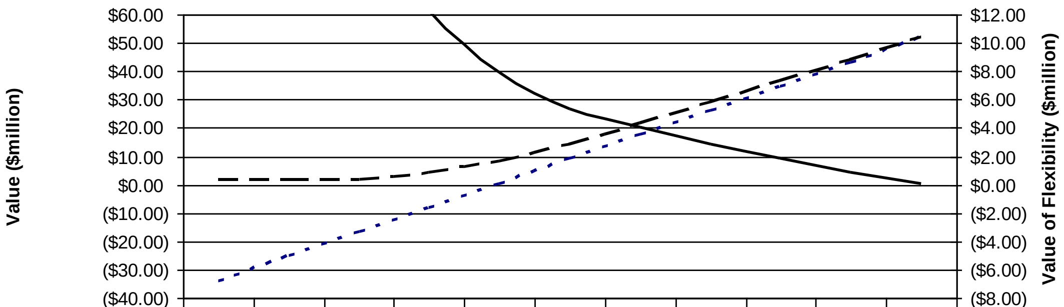
	$300	$350	$400	$450	$500	$550	$600	$650	$700	$750	$800
NPV (8yr)	($35.97)	($19.81)	($3.64)	$12.53	$28.70	$44.87	$61.03	$77.20	$93.37	$109.54	$125.71
ROV (8yr)	$1.75	$1.75	$5.51	$15.72	$29.93	$45.42	$61.34	$77.40	$93.53	$109.68	$125.84
Flex Val	$37.72	$21.56	$9.15	$3.19	$1.23	$0.56	$0.31	$0.20	$0.16	$0.14	$0.13

Price (A$/oz)

NPV (8yr) — ROV (8yr) — Flex Val

Figure 5 – Real Option Value – Cut-off Grade Problem

Figure 5 - Real Option Value - Cut-off Grade Problem



	$300	$350	$400	$450	$500	$550	$600	$650	$700	$750	$800
NPV (5yr)	($33.78)	($25.22)	($16.66)	($8.10)	$0.46	$9.03	$17.59	$26.15	$34.71	$43.27	$51.83
ROV (5yr)	$1.75	$1.75	$1.75	$4.18	$8.36	$14.37	$21.64	$29.02	$36.51	$44.11	$51.94
Flex Val	$35.53	$26.97	$18.41	$12.28	$7.89	$5.34	$4.05	$2.87	$1.80	$0.84	$0.10

Price (A$/oz)

NPV (5yr) — ROV (5yr) — Flex Val

As price increases, the need for management flexibility to change grade, mothball or abandon reduces. The real option value approaches the NPV and the value of management flexibility reduces. That is, as the project gets further in the money the real option value approaches the NPV.

The examples highlighted in this paper use exponential price increases. Use of the mean reverting prices outlined earlier in the paper is also possible and gives a wider applicability where commodity prices are not expected to increase exponentially.

Use of a spreadsheet to calculate option value using this algorithm is somewhat cumbersome due to the large amount of time necessary to set up the problem. This paper has demonstrated, however, that there is potential to use this algorithm to produce a practical valuation tool.

REFERENCES

Anderson T.J., 2000, "Real Options Analysis in Strategic Decision Making: an applied approach in a dual options framework.", *Journal of Applied Management Studies*, Vol. 9, No. 2, 2000, pp235-255.

Ballard J., 1994, "A Practitioners View of DCF Methods in Mineral Valuation", *Proceedings of VALMIN 94, AusIMM and MICA*, Sydney, Australia, Oct 1994, pp 37-45.

Butler P., 1994, "A Banker's View of Cash Flow Methods in Mineral Valuation", *Proceedings of VALMIN 94, AusIMM and MICA*, Sydney, Australia, Oct 1994, pp 47-62.

Heffernan V., 2004, "Mineral Property Valuation", *Engineering and Mining Journal*, August 2004, pp 21-26.

Herath H. S. B. and Park C. S., 2001, "Real Options Valuation and Its Relationship to Bayesian Decision-Making Methods", *The Engineering Economist*, Vol. 46 No. 1 2001.

Hull J. C., 1993, "Options, Futures, and Other Derivative Securities", *2nd edition, Prentice-Hall*, New Jersey, USA, 1993.

Laughton D. G. and Jacoby D., 1993, "Reversion, Timing Options, and Long-Term Decision-Making", *Financial Management*, Autumn 1993, pp225-240.

Lawrence M. J., 2001, "An Outline of Market-Based Approaches for Mineral Valuation Best Practice", *Mineral Asset Valuation Issues for the Next Millenium 2001 – Valmin 01 Conference, MICA*, Sydney October 2001, pp115-137.

Lonergan W. R., 1994, "The Financial Envelope – The Valuation of Securities after a Technical Evaluation", *Proceedings of VALMIN 94, AusIMM and MICA*, Sydney, Australia, Oct 1994, pp 225-235.

Luehrman T. A., 1997, "What's It Worth? A General Manager's Guide to Valuation", *Harvard Business Review*, May – June 1997, pp132-142.

O'Connor C. and McMahon D., 1994, "A Producing Miner's View of DCF Methods in Mineral Valuation", *Proceedings of VALMIN 94*, *AusIMM and MICA*, Sydney, Australia, Oct 1994, pp 75-80.

Park C. S. and Herath H. S. B., 2000, "Exploiting Uncertainty – Investment Opportunities as Real Options: A New Way of Thinking in Engineering Economics", *Engineering Economist*, Spring 2000 V45 i1 p1.

Robel G., 2001, "Real Options and Mean-Reverting Prices", *5th Annual Real Options Conference*, 2001 Los Angeles Anderson School of Management, UCLA.

Runge I. C., 1994, "Uncertainty and Risk in Mineral Valuation – A User's Perspective", *Proceedings of VALMIN 94, AusIMM and MICA*, Sydney, Australia, Oct 1994, pp 119-138.

Sorentino C., 2000, “Valuation Methodology for VALMIN”, *MICA, The Codes Forum*.

Stanford GSB, modified by Chan T., 1974, “Data General User’s Group Program Library – Business School Programs”, *Stanford Graduate School of Business*, July 1974, pp 273-280.

Taha H.A., 1976, “Operations Research – An Introduction”, *Macmillan Publishing Co.,* New York, 1976.

Trigeorgis L., 1993, “Real Options and Interactions With Financial Flexibility”, *Financial Manngement*, Autumn 1993, pp 202-224.

Wagner H.M., 1969, “Principles of Operations Research – with applications to Managerial Decisions”, *Prentice-Hall*, 1969.

Winsen J. K., 1994, “Project NPV as a Portfolio of Derivative Securities”, *Proceedings of VALMIN 94, AusIMM and MICA*, Sydney, Australia, Oct 1994, pp 103-117.

APPENDIX

This appendix provides an overview of the spreadsheet used to calculate the algorithm described in this paper. The spreadsheet used to evaluate the cut-off grade is presented here. The differences between the two spreadsheets are only minor.

The spreadsheet consists of three worksheets and two macros:

- A parameters sheet used to define constants for the problem;
- A binomial price tree;
- A binary valuation tree;
- A linear programming function; and,
- A macro to control the algorithm.

The parameters for both problems were similar. These parameters are shown in Exhibit 1. The problem is similar to that described by Winsen, 1994. While the parameters are held constant throughout the mine life, it is a simple matter to vary these parameters on an annual basis.

The binomial price tree is shown in Exhibit 2. This price tree and its associated probabilities gives an exponentially increasing price necessary for risk neutralized options valuation. It is a simple matter to use a mean reverting price tree and probabilities as described in the text for other pricing scenarios.

The binary valuation tree is shown in Exhibit 3. Each node contains the linear programming problem, including remaining reserves, mining rates and the selected grade. They also contain values for mothballing, closure (abandon) as well as the selection of the optimal course of action at each node. Only four years of mine life are shown in the diagram.

The case shown in Exhibit 3 is for a $500 gold price. It demonstrates the optimal management decisions made at various nodes based on the price, past decisions and possible future decisions. The initial decision is to mothball the operation until the price increases. Further price falls may lead to closure in years 3 or 4. Grades processed are a combination of medium and high grade depending on the remaining reserve, prices and costs in order to maximise the mine value.

EXHIBIT 1: GOLD MINE PARAMETERS Cut-off Grade Problem

S = $300 - 800 per oz. AUD

ore throughput:			max. available		max. life	
Q1 =	450,000	tonnes p.a.	1,211,956.52	tonnes	2.69	years
Q2 =	503,000	tonnes p.a.	1,858,333.33	tonnes	3.69	years
Q3 =	557,500	tonnes p.a.	2,787,500.00	tonnes	5.00	years

density = 2 tonnes/cubic metre

waste (stripping) ratios:

5.9 :1
3.5 :1
2 :1

material mined multiples: (1+waste ratio)/density

lambda1 =	3.45	cubic metres/tonne
lambda2 =	2.25	cubic metres/tonne
lambda3 =	1.5	cubic metres/tonne

recovered grades:

4.1250 grammes/tonne
3.5000 grammes/tonne
2.7500 grammes/tonne

x1 =	0.1326	oz/tonne (4.125/31.10347)
x2 =	0.1125	oz/tonne
x3 =	0.0884	oz/tonne

variable costs:

v1 =	$6.00	per cm (mining cost)
v2 =	$26.00	per tonne (milling cost)

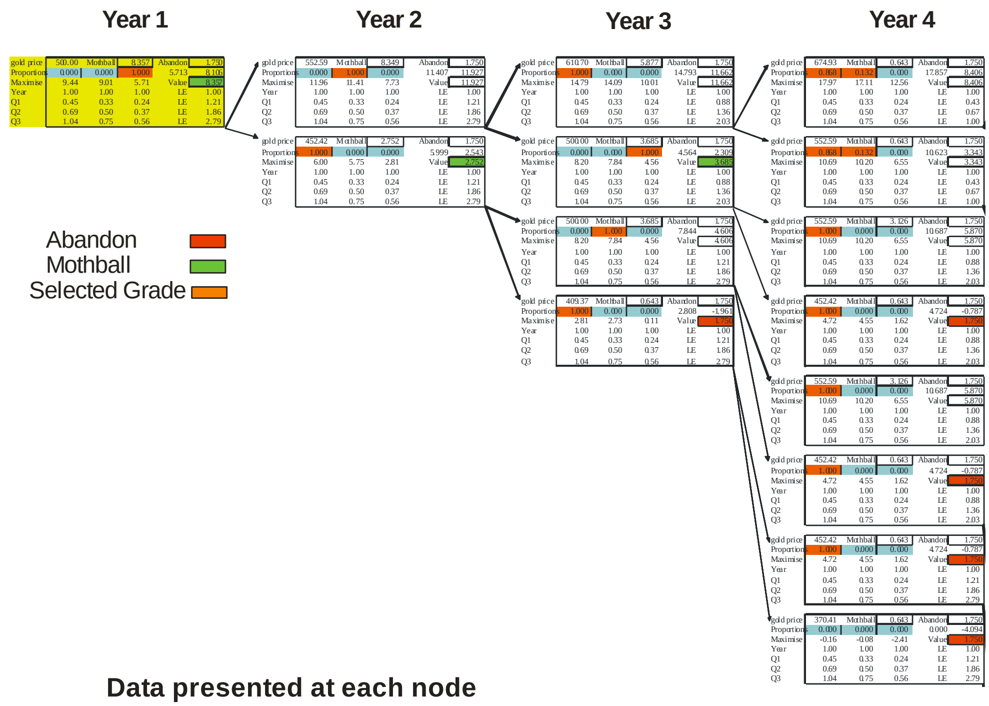
fixed costs\factors:

F1 =	$7,500,000	p.a. fixed costs
F2 =	$1,000,000	p.a. mothballing costs
A =	$1,750,000	salvage value
K/ =	$6,000,000	capital expansion cost
F/ =	$3,850,000	extra fixed costs

EXHIBIT 2: Price Tree

Year 1	Year 2	Year 3	Year 4	Year 5
				745.91
gold			674.93	
price		610.70		610.70
tree	552.59		552.59	
$500		500.00		500.00
	452.42		452.42	
		409.37		409.37
			370.41	
				335.16

Exhibit 3 - Binary Valuation Tree



Data presented at each node